Kubo number and magnetic field line diffusion coefficient for anisotropic magnetic turbulence

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(Received 27 November 2000; published 22 May 2001)

The magnetic field line diffusion coefficients D_x and D_y are obtained by numerical simulations in the case that all the magnetic turbulence correlation lengths l_x, l_y, and l_z are different. We find that the variety of numerical results can be organized in terms of the Kubo number, the definition of which is extended from $R=(\delta B/B_0)(l_\parallel/l_\perp)$ to $R=(\delta B/B_0)(l_z/l_x)$, for $l_x \geqslant l_y$. Here, $l_\parallel$ ($l_\perp$) is the correlation length along (perpendicular to) the average field $\mathbf{B}_0=B_0\hat{\mathbf{e}}_z$. We have anomalous, non-Gaussian transport for $R \lesssim 0.1$, in which case the mean square deviation scales nonlinearly with time. For $R \gtrsim 1$ we have several Gaussian regimes: an almost quasilinear regime for $0.1 \lesssim R \lesssim 1$, an intermediate, transition regime for $1 \lesssim R \lesssim 10$, and a percolative regime for $R \gtrsim 10$. An analytical form of the diffusion coefficient is proposed, $D_i=\mathcal{D}(\delta B l_z/B_0 l_x)^\mu (l_i/l_x)^\nu l_x^2/l_z$, which well describes the numerical simulation results in the quasilinear, intermediate, and percolative regimes.

DOI: 10.1103/PhysRevE.63.066405 PACS number(s): 52.25.Fi, 02.50.Ey, 95.30.Qd, 05.45.−a

I. INTRODUCTION

Many laboratory and astrophysical plasmas are characterized by a well developed spectrum of magnetic fluctuations. This magnetic turbulence influences the plasma behavior in several ways. In particular, particle transport in the plane transverse to the average magnetic field depends on the field line random walk due to low frequency magnetic turbulence [1–6]. This transport is of interest for magnetic confinement devices in the laboratory [7–9] and for energetic particle propagation in the heliosphere [10–15], in planetary foreshocks [16], across the Earth's magnetopause [17], and in extragalactic jets [18].

Several transport regimes can be found for the magnetic field lines, depending on parameters like the magnetic fluctuation level $\delta B/B_0$, the correlation lengths of magnetic turbulence l_x, l_y, and l_z in the x, y, and z directions, respectively, the Fourier spectrum model [6,11], and the dimensionality of turbulence [11,19–26]. Recently, Pommois *et al.* [23] (hereinafter, Paper I) found by means of a numerical study that transport can be anomalous, i.e., superdiffusive or subdiffusive, for low fluctuation levels $\delta B/B_0$; these anomalous regimes are related to the existence of closed magnetic surface for low levels of stochasticity. Also, transport is Gaussian (diffusive) for $\delta B/B_0 \gtrsim 0.2$, at least in the isotropic case characterized by $l_x=l_y=l_z$ (the exact definition of the correlation lengths is given in Sec. II). On the other hand, significant anisotropy in the distribution of magnetic turbulence power spectral density is found in many plasmas. In Paper I it was shown that in the case of anisotropy in the plane perpendicular to the average field $\mathbf{B}_0=B_0\hat{\mathbf{e}}_z$, the Gaussian regime was reached for larger values of $\delta B/B_0$ the larger l_x/l_y, with $l_x/l_y>1$; indicating by $(\delta B/B_0)^*$ the threshold of the fluctuation level to have Gaussian diffusion, the approximate proportionality $(\delta B/B_0)^* \propto l_x/l_y$ was found (see Fig. 7 of Paper I). Further, in the Gaussian regime, the diffusion coefficients D_x and D_y in the x and y directions were found to be roughly proportional to the corresponding correlation lengths l_x and l_y, that is, $D_x/D_y \sim l_x/l_y$ (see Fig. 9 of Paper I). Considering field line transport in the case of anisotropy in axially symmetric turbulence, that is, when the correlation length parallel to $\mathbf{B}_0$, $l_\parallel \equiv l_z$, is different from the correlation length perpendicular to $\mathbf{B}_0$, $l_\perp \equiv l_x = l_y$, Zimbardo *et al.* [24] (hereinafter, Paper II) found that the Gaussian regime is reached for lower values of $\delta B/B_0$ the larger $l_\parallel/l_\perp$, with $(\delta B/B_0)^* \propto l_\perp/l_\parallel$ (see Fig. 5 of Paper II), and that the various transport regimes are conveniently classified in terms of the Kubo number R [27,28]. For magnetic turbulence, the Kubo number can be defined as $R=(\delta B/B_0)(l_\parallel/l_\perp)$ (see, e.g., Refs. [29,8]). In Paper II it is found that for $R \lesssim 0.2$ there are anomalous, non-Gaussian transport regimes, which are considered in more detail in Refs. [20,22,23]. For $0.2 \lesssim R \lesssim 1$ there is an approximately quasilinear Gaussian diffusion regime. In the strictly quasilinear regime, the magnetic field line diffusion coefficient should scale as

$$D_\perp \propto \left(\frac{\delta B}{B_0}\right)^2 l_\parallel = R^2 \frac{l_\perp^2}{l_\parallel}; \tag{1}$$

actually, the scaling of $D_\perp$ with R found in Paper II is somewhat slower than R^2 (see Fig. 7 of Paper II). For $R \gtrsim 10$ an approximately percolative Gaussian diffusion regime [9] is found, in which

$$D_\perp \propto \left(\frac{\delta B}{B_0}\right)^{0.7} \frac{{l_\perp}^{1.3}}{{l_\parallel}^{0.3}} = R^{0.7} \frac{l_\perp^2}{l_\parallel}. \tag{2}$$

Here, $D_\perp = D_x + D_y$. For $1 \lesssim R \lesssim 10$ a transition regime is found (also Gaussian). Except for the anomalous transport regimes found for $R \lesssim 0.2$, these numerical findings are in agreement with the analytical results for the quasilinear [1–5] and percolative [8] regimes. In particular, the existence of the percolative regime was confirmed numerically by Ottaviani [19] and by Reuss and Misguish [30], and analytically by Vlad *et al.* [21] and by Milovanov [31]. Also, a nonquasilinear scaling of the diffusion coefficient with the fluctuation level was found by Gray *et al.* [11] with a two component magnetic turbulence model. We note that the values of the Kubo number R that separate one transport regime from another are approximate, and consequently we use the symbol $\lesssim$ to identify the intervals of R. On the one hand, this is due to the fact that the transition from one regime to the next is gradual. On the other hand, the numerical results also depend on the numerical representation of turbulence; features like the spectral extension and the spectrum model may influence the values of R that mark the transition from one regime to another.

Now, in many plasmas all three correlation lengths are different, $l_x \neq l_y \neq l_z$, so that it would be interesting to combine the results of Paper I and Paper II in a single expression for the diffusion coefficient, at least for a given range of the Kubo number R. In particular, in the solar wind, where a high level of magnetic turbulence is found, $\delta B/B_0 \sim 0.5$–1 [32], the turbulence correlation lengths can be very different, with $l_x/l_y \sim 3$–10 and $l_x/l_z \sim 0.1$–10 [33]. In such cases, the Kubo number falls within both the quasilinear regime $0.1 \lesssim R \lesssim 1$ and the intermediate transition regime $1 \lesssim R \lesssim 10$. Also, it would be interesting to understand whether the so called Bohm scaling of the diffusion coefficient, in which $D_\perp \propto R$, can be reproduced by numerical simulations, for instance, in the intermediate regime $1 \lesssim R \lesssim 10$, since the existence of this regime has been questioned in Refs. [21,30].

In this paper we extend the numerical simulations of Paper I and Paper II to the case of ''general'' anisotropy $l_x \neq l_y \neq l_z$ (with some regard to the solar wind turbulence to identify the relevant range of parameters), and propose an approximate, analytic form of the diffusion coefficient valid for the Gaussian regimes characterized by $R \gtrsim 0.1$. To this end, the definition of the Kubo number has to be extended to the case $l_x \neq l_y$. As discussed later, we find that $l_\perp$ can be substituted by l_x, with the understanding that l_x is the largest of the correlation lengths in the plane perpendicular to $\mathbf{B}_0$. Then the parameters of the analytical model are fitted to the numerical results for the quasilinear, the intermediate, and the percolative regimes, yielding a satisfactory description of the field line diffusion coefficient for R varying over three decades.

II. NUMERICAL STUDY

Following Papers I and II, we trace the magnetic field lines by integrating the equation

$$\frac{d\mathbf{r}}{ds} = \frac{\mathbf{B}(\mathbf{r})}{|\mathbf{B}(\mathbf{r})|}, \tag{3}$$

where s is the field line length, and the magnetic field at a location $\mathbf{r}$ is $\mathbf{B}(\mathbf{r}) = \mathbf{B}_0 + \delta\mathbf{B}(\mathbf{r})$. Here, $\mathbf{B}_0 = B_0 \hat{\mathbf{e}}_z$, and the magnetic perturbation $\delta\mathbf{B}(\mathbf{r})$ is represented as the sum of static magnetic perturbations [20]:

$$\delta\mathbf{B}(\mathbf{r}) = \sum_{\mathbf{k},\sigma} \delta B(\mathbf{k}) \hat{\mathbf{e}}_\sigma(\mathbf{k}) \exp i[\mathbf{k}\cdot\mathbf{r} + \phi_{\mathbf{k}}^\sigma], \tag{4}$$

where $\delta B(\mathbf{k})$ is the Fourier amplitude of the mode with wave vector $\mathbf{k}$ and polarization σ, $\hat{\mathbf{e}}_\sigma(\mathbf{k})$ are the polarization unit vectors, and $\phi_{\mathbf{k}}^\sigma$ are random phases. The Fourier amplitude of the modes is given by the spectrum $\delta B(\mathbf{k}) \propto 1/(k_x^2 l_x^2 + k_y^2 l_y^2 + k_z^2 l_z^2)^{\gamma/4+1/2}$, where $\gamma = 3/2$ is the spectral index. The wave vectors are taken on a discrete grid and satisfy $k_x^2 l_x^2 + k_y^2 l_y^2 + k_z^2 l_z^2 \geqslant 4\pi^2$ (see Refs. [22,23] for more details). Therefore, in each direction the longest wavelength present in the turbulence model corresponds to the correlation length. (Because of anisotropy, the simulation box is not cubic but is a parallelepiped.) The correlation lengths also determine the shape of the ellipsoid representing the wave-vector distribution in $\mathbf{k}$ space. Equation (3) is integrated numerically to yield the field line position for a large number of randomly chosen starting conditions. The transport of magnetic field lines is analyzed with the generalized diffusion law,

$$\langle \Delta x_i^2 \rangle = 2 D_i s^{\alpha_i}, \quad i = (x,y). \tag{5}$$

When the anomalous diffusion exponent $\alpha_i \simeq 1$, we have normal Gaussian diffusion, when $\alpha_i < 1$ we have subdiffusion, and when $\alpha_i > 1$ we have superdiffusion. In this paper we report the diffusion coefficients only in the Gaussian regime, that is, when $\alpha_i = 1 \pm 0.1$, and, at the same time, the value of the kurtosis $K_i = \langle \Delta x_i^4 \rangle / (\langle \Delta x_i^2 \rangle)^2$ differs from the Gaussian value of 3 by less than 10%. The fitting procedure involves two steps, that is, first we check that we are in the Gaussian regime by a calculation of the kurtosis and a fit of Eq. (5) to calculate α_i; then a second fit is realized setting $\alpha = 1$ in Eq. (5), to reevaluate the diffusion coefficients D_i. With this procedure we obtain a reduction of the statistical error on the diffusion coefficients, as these are not influenced by the errors on α_i. In Fig. 1, D_x and D_y are given as functions of $\delta B/B_0$ for various degrees of anisotropy, which are quantified by the ratio of the correlation lengths and are indicated in the figure caption. The field line diffusion coefficients have the dimensions of a length, and are normalized with respect to l_y, which is not changed in the numerical simulations, so that the plotted quantities are dimensionless. For clarity, D_x and D_y are plotted on two different panels. It can be seen that very different values can be obtained, spanning three decades. For a given value of $\delta B/B_0$, e.g., 0.4–0.7, the diffusion coefficient varies by 1.5 orders of magnitude when changing the correlation length ratios. For small or moderate values of $\delta B/B_0$ some diffusion coefficients are not given, as transport falls into the anomalous regimes.

Unless one is interested in a single set of values of l_x, l_y, l_z, and $\delta B/B_0$, it is not easy to extract the required information on transport from these results. For instance, when moving either across the Earth's magnetopause or for large distances in the heliosphere, the above parameters change continuously. Therefore, it would be useful to find an analytical expression for the diffusion coefficients which can approximately represent the results of the numerical simulation.

III. GENERAL EXPRESSION FOR THE DIFFUSION COEFFICIENT IN THE GAUSSIAN REGIMES

The expressions given in Eqs. (1) and (2) are the typical forms for $D_\perp$ in the two limiting regimes, that is, for very

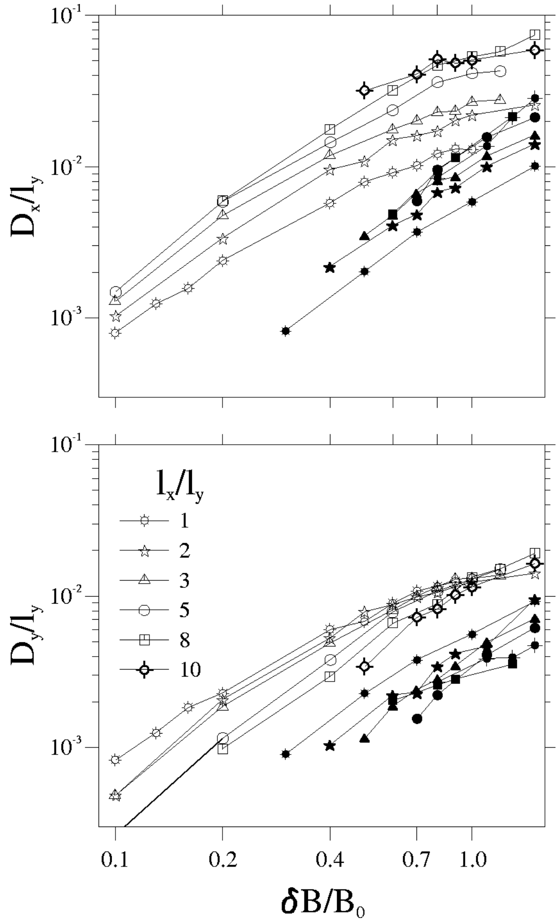

FIG. 1. Coefficient D_x and D_y versus fluctuation level $\delta B/B_0$, and for different values of the correlation lengths l_x, l_y, l_z. Dimensionless units. The degree of anisotropy l_x/l_y in the xy plane is represented by different symbols. Suns, $l_x/l_y=1$; stars, $l_x/l_y=2$; triangles, $l_x/l_y=3$; circles, $l_x/l_y=5$; squares, $l_x/l_y=8$; crosses-circles, $l_x/l_y=10$. Solid markers, $l_z/l_y=1$; open markers, $l_z/l_y=10$.

small and very large values of the Kubo number. It can be seen that what changes is, basically, the value of the exponent of R. This suggests that a suitable expression for the diffusion coefficient in the intermediate regime should be of the form $D_\perp \propto R^\mu l_\perp^2/l_\parallel$, where μ is a parameter between 0.7 and 2 to be determined. Moreover, the required form of the diffusion coefficient should be able to describe as well situations where there is anisotropy in the plane perpendicular to the mean magnetic field $\mathbf{B}_0$. Thus, l_x and l_y should explicitly appear in the required expression, and we have to redefine $l_\perp$ and the Kubo number R. In our simulations we consider, without loss of generality, the x direction as the direction where the correlation length is larger in the plane perpendicular to $\mathbf{B}_0$ ($l_x \geqslant l_y$). When $l_x \gg l_y$, the wave vectors are squeezed along y and, since the magnetic fluctuations are transverse, most turbulence energy is along x (see Paper I). Therefore, l_x is the most significant correlation length in the xy plane, and we will assume that $l_\perp \equiv l_x$. Then the Kubo number becomes $R \equiv (\delta B/B_0)(l_z/l_x)$, as $l_\parallel \equiv l_z$. Since the level of stochasticity depends on the Kubo number (Paper II), the position $l_\perp \rightarrow l_x$ allows us to understand the fact, reported in the Introduction and in Paper I, Fig. 7, that the Gaussian regime (corresponding to global stochasticity) is reached for higher values of $\delta B/B_0$ when l_x/l_y is increased, with l_z kept constant. Therefore, the results of Paper I support our choice for $l_\perp$ and for the Kubo number. Other forms of $l_\perp$ have been tested, like $l_\perp=\sqrt{(l_x^2+l_y^2)/2}$ and $l_\perp=\sqrt{l_x l_y}$, but the corresponding fit of the diffusion coefficients, reported below, was less good.

TABLE I. Parameters obtained in the least squares fit of the diffusion coefficients with Eq. (6).

Range of R	$\mathcal{D}$	μ	ν	χ_r^2
$0.1 \leqslant R \leqslant 1$	0.0301	1.67	0.78	2.6
$1 \leqslant R \leqslant 10$	0.0358	1.19	0.82	1.8
$R>10$	0.125	0.81		2.4
All cases	0.0270	1.34	0.82	6.7

Further, we have to consider the influence of the anisotropy l_x/l_y on the magnetic field line transport. We found in Paper I an almost linear relation between the diffusion coefficient ratio and the correlation length ratio: $D_x/D_y \sim (l_x/l_y)^\nu$, where ν should be approximatively 1. Thus an expression for the diffusion coefficient D_i similar to Eqs. (1) and (2) and that takes into account all the characteristics discussed above, might be the following:

$$D_i=\mathcal{D}\left(\frac{\delta B}{B_0}\frac{l_z}{l_x}\right)^\mu\left(\frac{l_i}{l_x}\right)^\nu\frac{l_x^2}{l_z}, \tag{6}$$

where $i=x,y$, and $\mathcal{D}$, μ, and ν are dimensionless parameters to be determined. Of course, the quantities in the first parentheses correspond to the Kubo number just defined, $R=(\delta B/B_0)(l_z/l_x)$. In the second parentheses, giving the dependence of D_i on l_i^ν, we divided l_i by l_x in order to have a dimensionless factor. From the ratio of the correlation lengths in Eq. (6), a spare l_x remains which is to be used to set the physical value of the diffusion coefficients through comparison with the turbulence correlation lengths of the problem under consideration.

We present in Table I the parameters $\mathcal{D}$, μ, and ν of Eq. (6) obtained with a least squares fit to the diffusion coefficients reported in Fig. 1. To fit independently each of the different regimes, we grouped the diffusion coefficients according to the Kubo number and made three subsets of data: first, diffusion coefficients in the quasilinear regime with $0.1 \leqslant R \leqslant 1$; second, diffusion coefficients in the intermediate regime with $1 \leqslant R \leqslant 10$; third, diffusion coefficients in the percolative regime with $R>10$. As an indication of the goodness of the fit we write in the last columm of the table the χ_r^2 obtained in the fit (the reduced χ_r^2 is evaluated assuming 15% of error on all the diffusion coefficients; this error corresponds to the statistical uncertainity on the values of D_x and D_y reported in Fig. 1). We also made a least squares fit to other analytical forms of the diffusion coefficient, different from Eq. (6). However, the fit was less good, as indicated by χ_r^2.

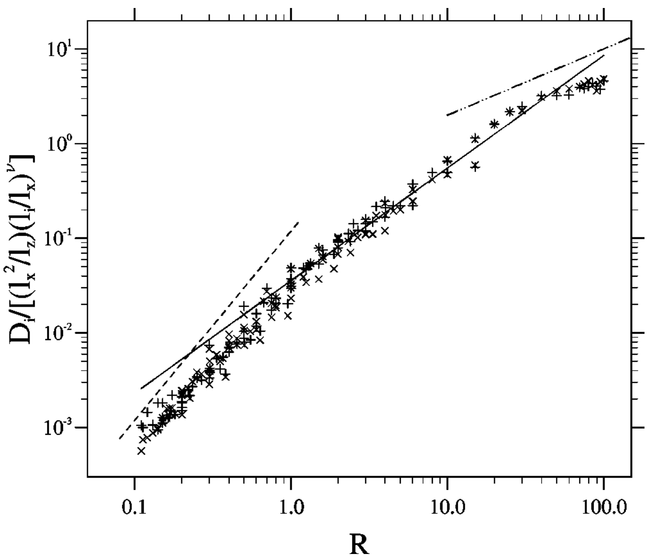

FIG. 2. Ratio $D_i/[(l_x^2/l_z)(l_i/l_x)^\nu]$ versus Kubo number R, and best fitting line for the data having $1 \leqslant R \leqslant 10$ (solid line). This line is extended over the whole range of R for comparison with the other scalings. The dashed line represents the quasilinear scaling and the dash-dotted line the percolative scaling. Dimensionless units. Symbols: plus signs for D_x; crosses for D_y.

For the cases having $0.1 \leqslant R \leqslant 1$, we obtain $\mu = 1.67$, which is smaller than 2, as it should be in the quasilinear regime. This discrepancy is due to the fact that the quasilinear regime is obtained in the limit of small Kubo number, for $R \ll 1$, not for $0.1 \lesssim R \lesssim 1$. On the other hand, for $R \lesssim 0.1$ we find anomalous diffusion ($\alpha_i \neq 1$) in our simulations, and thus the diffusion constant D_i in Eq. (5) cannot be compared to diffusion coefficients. Anyway, for the sake of brevity we still call the regime with $0.1 \lesssim R \lesssim 1$ quasilinear. Also, $\nu = 0.78$, which is rather close to 1, as anticipated. In the intermediate regime, $1 \leqslant R \leqslant 10$, we obtain $\mu = 1.19$, as expected, a value of μ between 0.7 and 2, and $\nu = 0.82$, which is close to 1. While the value of μ shows a clear transition from $R \leqslant 1$ to $R \geqslant 1$, the value of ν is nearly unchanged, which implies that the effect of anisotropy in the plane perpendicular to $\mathbf{B}_0$ is the same in the quasilinear and the intermediate regimes. Morever, when fitting separately D_x or D_y, the fitted parameters change very little, showing that independent subsets of data lead to the same representation of the diffusion coefficients. For $R > 10$ we obtain $\mu = 0.81$, which is rather close to the theoretical prediction for the percolative regime, $\mu = 0.7$. We may argue that simulations with $R > 100$ should fully confirm the percolation scaling. On the other hand, only few runs with $l_x/l_y \neq 1$ were done in this regime, so that the determination of ν is not feasible and is not reported. Finally, a fit to the diffusion coefficient of all the runs yields $\mu = 1.34$ and $\nu = 0.82$; in this case the χ_r^2 is much larger than in the previous cases.

To better appreciate the significance of the above results for μ and ν we plot in Fig. 2 the ratio $D_i/[(l_x^2/l_z)(l_i/l_x)^\nu]$ versus the Kubo number, for all of the 111 cases that are reported in Fig. 1. We use the value of $\nu = 0.82$, as this is almost the same for all the regimes (see Table I). We also plot in Fig. 2 the best fitting line for the cases having $1 \leqslant R \leqslant 10$, with a slope corresponding to the exponent $\mu = 1.19$. Although the cases shown in the figure span three decades of R, and four decades of $D_i/[(l_x^2/l_z)(l_i/l_x)^\nu]$, the data points follow rather well a piecewise straight line throughout all the range of R, showing that the mathematical form proposed for the diffusion coefficient is suitable for the quasilinear, intermediate, and percolative diffusion regimes. In particular, the spread of the points representing D_x (plus signs) and D_y (crosses) about the fitting line is very limited, especially when compared to Fig. 1, and D_x and D_y appear to be fitted equally well.

For $0.1 \leqslant R \leqslant 1$ we indicated in Fig. 2 the quasilinear scaling, $\mu = 2$, by the dashed line, slightly upshifted with respect to the data. It appears that the data point almost follow such a scaling for $0.1 \leqslant R \leqslant 1$, although the slope is somewhat lower, as indicated in Table I. Conversely, for $R \gtrsim 10$, the points are aligned with the dash-dotted line, which represents the percolative scaling characterized by $\mu = 0.7$ [9,24]. Also, values of μ smaller than 0.7 have been predicted for $R \gg 1$ [21,31], and indeed the slope of the rightmost points in Fig. 2 appears to be less than that of the dash-dotted line.

Finally, we note that for some problems involving magnetic field line transport it may be preferable to write the equations for the magnetic field lines as

$$\frac{d\mathbf{r}}{d\xi} = \frac{\mathbf{B}(\mathbf{r})}{|\mathbf{B}_0|} = \frac{\mathbf{B}_0 + \delta\mathbf{B}(\mathbf{r})}{|\mathbf{B}_0|}. \tag{7}$$

Here, ξ has the dimensions of a length and is related to the field line length s through $ds/d\xi = |\mathbf{B}|/|\mathbf{B}_0|$. Indeed, ξ represents the length of the unperturbed field lines, i.e., for $\delta\mathbf{B}(\mathbf{r}) = 0$. This form of the field line equations is appropriate for problems where the field line transport is studied by a Monte Carlo simulation; see, e.g., [14,15]. From Eq. (7), a new set of diffusion coefficients is obtained as a function of l_x, l_y, and l_z [14]. Note that we always have $s > \xi$, as the magnetic field lines meandering because of turbulence are longer than the unperturbed field lines. Correspondingly, the diffusion coefficients found with Eq. (7) are somewhat larger than those reported in this paper. Still, the coefficients obtained are fitted equally well by Eq. (6), and in particular for $0.1 \lesssim R \lesssim 10$, which is of interest for the solar wind, we find $\mathcal{D} \simeq 0.03$, $\mu \simeq 1.5$, and $\nu \simeq 0.7$.

IV. DISCUSSION AND CONCLUSIONS

In this paper we have looked for an analytical form of the magnetic field line diffusion coefficients D_x and D_y that could describe, within a reasonable approximation, our numerical results for the diffusion coefficient in the case that all the correlation lengths l_x, l_y, and l_z are different. To this end, in the case of anisotropy in the plane perpendicular to $\mathbf{B}_0 = B_0\hat{\mathbf{e}}_z$, the transverse correlation length $l_\perp$ in the expression for the Kubo number is changed to the larger of l_x and l_y (in our simulations, $l_x \geqslant l_y$). Therefore, the definition of Kubo number is extended from $R = (\delta B/B_0)(l_\parallel/l_\perp)$ to $R = (\delta B/B_0)(l_z/l_x)$, for $l_x \geqslant l_y$. We find that the diffusion coefficients can be organized in terms of this generalized

Kubo number even when $l_x \neq l_y \neq l_z$, so that we have anomalous, non-Gaussian transport for $R \lesssim 0.1$, a Gaussian almost quasilinear regime for $R \lesssim 1$, an intermediate Gaussian regime for $1 \lesssim R \lesssim 10$, and a percolative Gaussian regime for $R \gtrsim 10$. These findings are in good agreement with the results of Paper II, where only $l_x = l_y$ was considered.

We have further proposed a form of the diffusion coefficient $D_i = \mathcal{D} R^{\mu} (l_i / l_x)^{\nu} l_x^2 / l_z$, which can describe well the numerical simulation results in the different regimes. We find that $\nu \simeq 0.8$ changes very little from one regime to another, while μ changes from $\mu = 1.67$ for $0.1 \lesssim R \lesssim 1$, to $\mu = 1.19$ for $1 \lesssim R \lesssim 10$, and to $\mu = 0.81$ for $R \gtrsim 10$. In any case, it appears that the so called Bohm scaling of the diffusion coefficient, corresponding to $\mu = 1$, is not recovered by the numerical simulations [21]. Such an analytical form of D_i is useful when it is necessary to know the value of the diffusion coefficients in situations where the same parameters $\delta B / B_0$ and l_x, l_y, and l_z are varying. This includes the transport of particles in the solar wind [14,15], across the magnetopause, and, in perspective, in plasma confinement devices where the turbulence level increases toward the edge of the plasma, as in reversed field pinches. Our study emphasizes the importance of knowing, besides the turbulence level, all three correlation lengths in the case of anisotropic turbulence in order to determine the Kubo number, the transport regime, and the diffusion coefficients.

ACKNOWLEDGMENTS

This work is part of a research program that is financially supported by the Ministero dell'Università e della Ricerca Scientifica e Tecnologica (MURST), the Consiglio Nazionale delle Ricerche (CNR) Contracts No. 98.00129.CT02 and No. 98.00148.CT02, the Agenzia Spaziale Italiana (ASI) Contract No. ARS98-82, and the INTAS Open Grant No. 97-1612.

[1] M.N. Rosenbluth, R.Z. Sagdeev, G.B. Taylor, and G.M. Zaslavsky, Nucl. Fusion **6**, 297 (1966).
[2] N.N. Filonenko, R.Z. Sagdeev, and G.M. Zaslavsky, Nucl. Fusion **7**, 253 (1967).
[3] J.R. Jokipii, Astrophys. J. **146**, 480 (1966).
[4] J.R. Jokipii and E.N. Parker, Phys. Rev. Lett. **21**, 44 (1968).
[5] A.B. Rechester and M.N. Rosenbluth, Phys. Rev. Lett. **40**, 38 (1978).
[6] W.H. Matthaeus, P.C. Gray, D.H. Pontius, Jr., and J.W. Bieber, Phys. Rev. Lett. **75**, 2136 (1995).
[7] J.A. Krommes, C. Oberman, and R.G. Kleva, J. Plasma Phys. **30**, 11 (1983).
[8] M.B. Isichenko, Plasma Phys. Controlled Fusion **33**, 795 (1991).
[9] M.B. Isichenko, Plasma Phys. Controlled Fusion **33**, 809 (1991).
[10] J. Kota and J.R. Jokipii, Science **268**, 1024 (1995).
[11] P.C. Gray, D.H. Pontius, and W.H. Matthaeus, Geophys. Res. Lett. **23**, 965 (1996).
[12] J. Giacalone, J. Jokipii, and J.E. Mazur, Astrophys. J. **532**, L75 (2000).
[13] J.E. Mazur, G.M. Mason, J.R. Dwyer, J. Giacalone, J.R. Jokipii, and E.C. Stone, Astrophys. J. **532**, L79 (2000).
[14] P. Pommois, P. Veltri, and G. Zimbardo, J. Geophys. Res. [Space Phys.] (to be published).
[15] G. Zimbardo, P. Pommois, and P. Veltri, Phys. Chem. (to be published).
[16] G. Zimbardo and P. Veltri, Geophys. Res. Lett. **23**, 793 (1996).
[17] R.A. Treumann, Geophys. Res. Lett. **24**, 1727 (1997).
[18] E. Corbelli and P. Veltri, Astrophys. J. **340**, 679 (1989).
[19] M. Ottaviani, Europhys. Lett. **20**, 111 (1992).
[20] G. Zimbardo, P. Veltri, G. Basile, and S. Principato, Phys. Plasmas **2**, 2653 (1995).
[21] M. Vlad, F. Spineanu, J.H. Misguich, and R. Balescu, Phys. Rev. E **58**, 7359 (1998).
[22] P. Pommois, G. Zimbardo, and P. Veltri, Phys. Plasmas **5**, 1288 (1998).
[23] P. Pommois, P. Veltri, and G. Zimbardo, Phys. Rev. E **59**, 2244 (1999) (Paper I).
[24] G. Zimbardo, P. Veltri, and P. Pommois, Phys. Rev. E **61**, 1940 (2000) (Paper II).
[25] G. Zimbardo, P. Pommois, and P. Veltri, Physica A **280**, 99 (2000).
[26] M. Ottaviani and M. Pettini, Int. J. Mod. Phys. B **5**, 1243 (1991).
[27] R. Kubo, J. Math. Phys. **4**, 174 (1963).
[28] A. Brissaud and U. Frisch, J. Math. Phys. **15**, 524 (1974).
[29] B.B. Kadomtsev and O.P. Pogutse, in *Plasma Physics and Controlled Nuclear Fusion Research, Proceedings of the Seventh International Conference* (Innsbruck, 1978) (IAEA, Vienna, 1979), Vol. 1, p. 649.
[30] J.-D. Reuss and J.H. Misguich, Phys. Rev. E **54**, 1857 (1996).
[31] A.V. Milovanov, Phys. Rev. E **63**, 047301 (2001).
[32] G.P. Zank, W.H. Matthaeus, and C.W. Smith, J. Geophys. Res., [Space Phys.] **101**, 17 093 (1996).
[33] V. Carbone, F. Malara, and P. Veltri, J. Geophys. Res. [Space Phys.] **100**, 1763 (1995).